Houghton Conquest
Lower School
Growing together

Written Calculation Policy – to match New National Mathematics Curriculum (In conjunction with the Mental Calculation Policy November 2015)

This document presupposes that you wish to teach calculation with understanding, and not just as a process that is to be remembered. The Written Calculation Policy clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports the development of mathematical concepts. When reading this document please refer the the New National Curriculum Appendix 1 for Mathematics.
https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/238967/Mathematics_Appendix_1.pdf

A separate Mental Calculation Policy outlines mental strategies, including the use of jottings, vocabulary to be developed and the key number facts that children will need to know. Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

The Aims of the Curriculum:

The National Curriculum for Mathematics aims to ensure that all pupils:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practise with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Recording

Recording is developed in a range of ways discussed below. Although they will be developed in this order, once a way of recording such as 'by showing real objects' is in place, that will continue to be used throughout the Primary years when needed. In the EYFS most recording will be demonstrated through the use of 'real' objects.

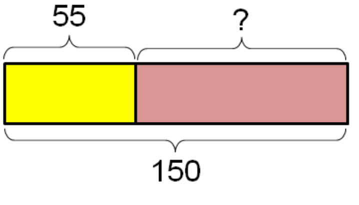
Development of recording:

- The use of real objects
- by photographing or drawing the calculation activity
- counting on a number line
- a practical calculation activity on a number line
- a number bond on a number line
- written calculations on a number line
- a practical activity as a number sentence
- a number bond as a number sentence
- written calculations as a number sentence
- written methods

Mastery
At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.
Children are encouraged to give answers in full sentences to help them fully understand the maths concepts they are learning.

Teaching for Mastery: White Rose Planning
These planning overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.
The overviews:
- have number at their heart. A large proportion of time is spent reinforcing number to build competency.
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group.
- provide plenty of time to build reasoning and problem solving elements into the curriculum.

Concrete – Pictorial – Abstract
- We believe that all students, when introduced to a key new concept, should have the opportunity to build competency in this topic by taking this approach.
- **Concrete** – students should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.
- **Pictorial** – students should then build on this concrete approach by using pictorial representations. These representations can then be used to reason and solve problems.
- **Abstract** – with the foundations firmly laid, students should be able to move to an abstract approach using numbers and key concepts with confidence.



An example of a bar modelling diagram used to solve problems.

Progression in calculation

Addition
Children begin calculation purely with practical activities using objects such as beads, playdough, bears and puzzles. Over time they learn to record these activities by explaining different methods such as using objects or mark making and then recording. Recording will be by showing or taking photographs of the equipment they have used, leading to making marks of what they did.

For instance, with the practical activity - I have 3 sweets, then I get one more. The child draws the sweets. They may draw 3 sweets and then another. They may just draw 4 to start with.

() () () ()

They won't draw 3, then 1, then 4, nor should they be expected to at this stage, but many chose to record using numerals depending on their stage of development.

() () () () = () () () () doesn’t make much sense. You either have 3 and 1 or you have 4. You never have both.

This means that any recording of the format 3+1=4 is very unhelpful and is not typically based on their experience but on an abstract recording method. However, some children may have grasped the concept and will naturally progress to using written methods.

When pupils are ready to record numerals (possibly during the summer term in reception but mainly in Year One) they may begin to record the above example with numbers as well:

() () ()   ()
3   **1**

or just as

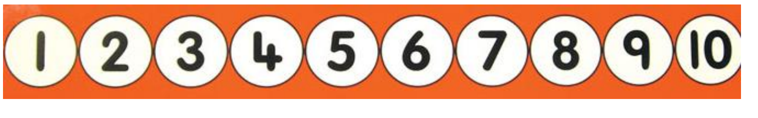
() () () ()
4

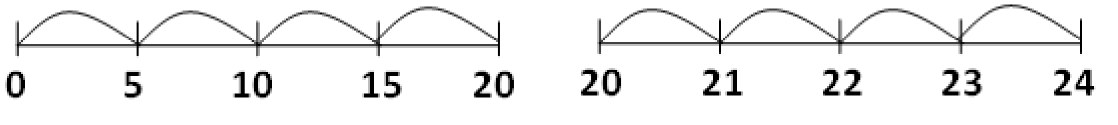
When the children are ready they will start to record their work in number sentences. For example:

3 + 1 = 4

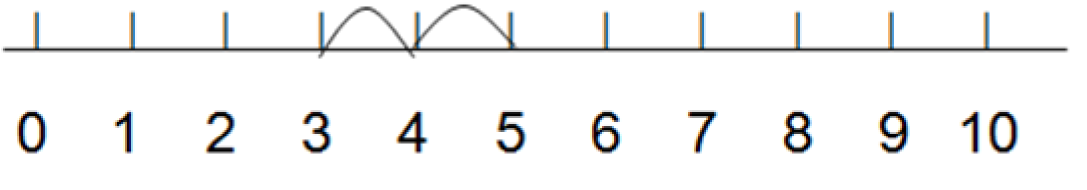
As well as using objects, pupils will begin to use number lines both as practical equipment that makes the calculation transparent and as a way to record what they did.

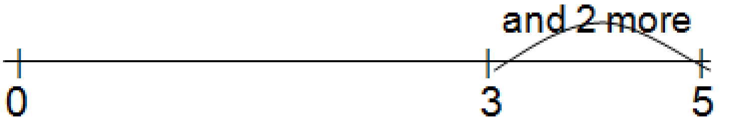
At first children will record their counting on number tracks, later moving to recording of calculation on a number line.





Pupils will use numbered number lines to record jumps, for example for 3 + 2, before recording on blank number lines.





By the end of Year One, children should be confident using number lines to ‘experiment’ with numbers.

Recording number sentences

Before pupils move to recording 3+1 they will need lots of experience of practical addition, and an ability to respond to mathematical vocabulary practically. For instance, if you ask a child to show you 5 and 2 more, or 3 plus 1, or 1 add 4, they can use teddies, counters or number tracks to show you. They will also be developing their use of mathematical vocabulary to explain what they have done.

24.5 + 87.8 = 20 + 80 + 4 + 7 + 0.5 + 0.8
= 100 + 11 + 1.3
= 112.3

From this it will be possible to develop an understanding of the + sign, which will enable pupils to begin to record in the form 5+2.

Pupils then need to understand the concept of equality before using the = sign. This means they can see an example such as 7=6+1, or 5=5, as well as the more common arrangement 3+1=4, and know that it makes sense.

In Year One pupils will still work practically with equipment and real objects, but now can record their explanation of what they have done as a conventional number sentence:

3 + 14 = 17 **17 = 14 + 3** **17 – 3 = 14** **3 + 14 = 14 + 3** and so on.

However, pupils will still record with objects, drawings and number lines **on a frequent basis**, and whenever they are learning new concepts or starting to use a wider range of numbers they will need to return to using these easily understood and explained methods of recording.

Partitioning

Once children have a secure understanding of place value in 2 digit numbers they start to use partitioning.
Partitioning may be recorded in a number of ways, such as:

The important thing to consider when children are recording partitioning is that they record how they thought about the numbers, and don’t all try to do it the same way. This is not about finding lots of ways to record, but of recording what makes sense to a child.

36 + 45 = 30 + 40 + 6 + 5
= 70 + 11
= 81

or

36 + 45 = 36 + 40 + 5
= 76 + 5
= 81

Partitioning is also an appropriate strategy for larger numbers, eventually including decimals.

536 + 245 = 500 + 200 + 30 + 40 + 6 + 5
= 700 + 70 + 11
= 781

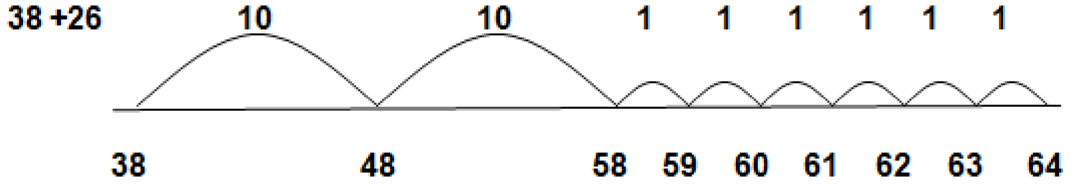
Partitioning using number lines

Key understanding – a number line is a tool, not

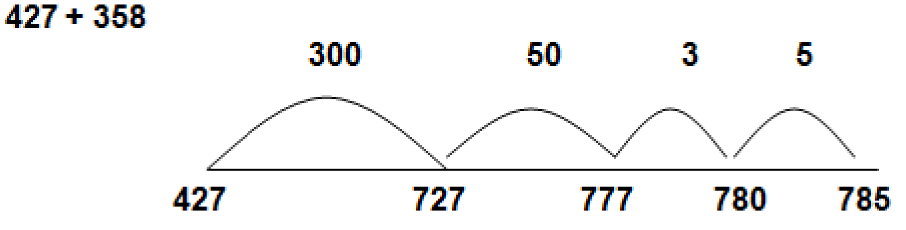
a rule.

Children partition numbers to count on, mainly in multiples of 100, 10 or 1, on a number line. Number lines will be used for calculations right through Key Stage 2.

Initial attempts may be a little slow as children choose easy numbers to count on:

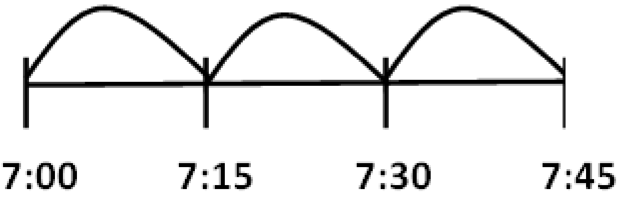


As they become more confident, children start to jump in multiples of 100, 10 and 1. They use their own choice of numbers, doing any jumps on the number line, in steps of 100, 10, 1 or multiples of these, depending on their ability and understanding of place value.

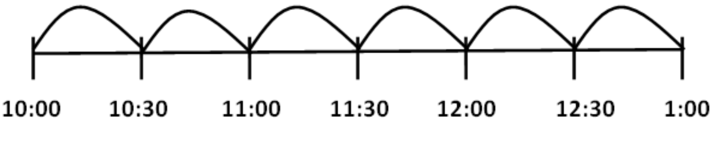


Children need to develop understanding of calculation in a range of contexts, for instance measures, including money and time.

Time is particularly difficult, and at first children will use number lines to record counting in steps of hours or minutes.



Counting across boundaries is particularly important.



Expanded Vertical Method

In Key Stage 2 pupils may begin to record addition calculations vertically, at first recording calculations both as the partitioning they have been using and as an expanded vertical calculation, adding numbers in columns, beginning with the ones, tens and then hundreds. The vocabulary used will always be whole number place value vocabulary, so 254 would be 200, 50 and 4, never 2 hundreds, 5 tens and 4 ones.

	5	3	6
+	2	4	5
		1	1
		7	0
	7	0	0
	7	8	1

Once pupils are very confident with this method of recording they may extend it to numbers with more digits, providing their understanding of place value is sufficient to support this.

	5	3	6	6
+	4	4	5	1
				7
		1	1	0
		7	0	0
	9	0	0	0
	9	8	1	7

	5	3	.	6
+	4	4	.	1
		0	.	7
		7	.	0
	9	0	.	0
	9	7	.	7

Compact method

It is possible to record the vertical method more quickly by making a note when the addition of two or more numbers goes above 1, 10 or 100 and so on, rather than writing it all out. This is known as the compact method.

When children use the compact method they need to know that it works in a similar way to partitioning, but that you add the ones first. It is important that children understanding the value of the digits before using this method.

789 + 642 becomes

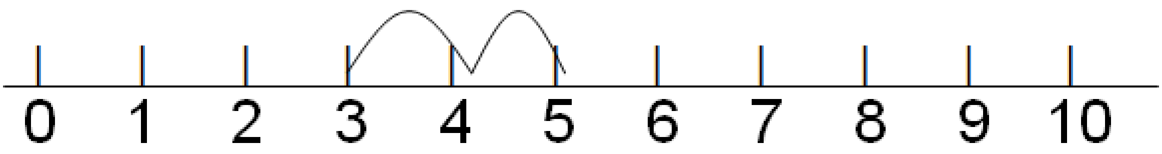
	7	8	9
+	6	4	2
1	4	3	1
	1	1	

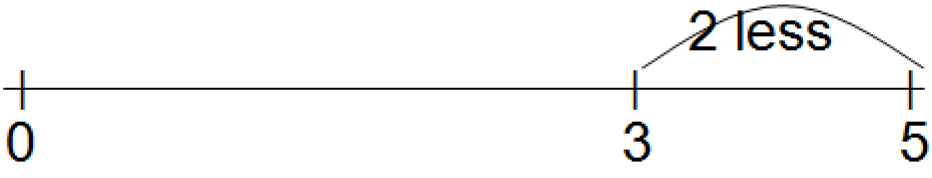
Answer: 1431

Subtraction

As with addition, subtraction is initially recorded as marks representing the result of a practical activity, moving on to record this using numbers, on number tracks or lines or as number sentences.

Initially number tracks or lines will be used to subtract small numbers such as 5 – 2.



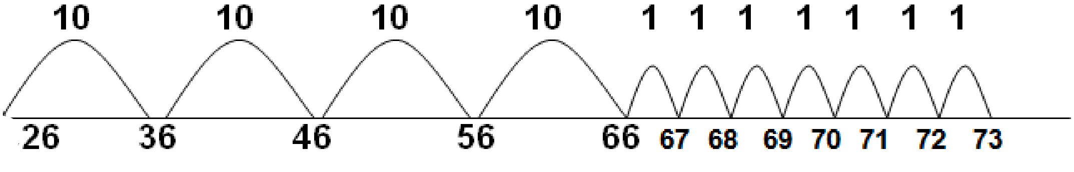


When pupils move on to use jottings the number line will become especially important. Jottings as number sentences are less useful for subtraction as partitioning cannot generally be used. In the example 73 – 26 it is possible to start with 70 – 20, but 3 – 6 is less useful!

Key understanding – Pupils need to realise that partitioning is not appropriate for subtraction.

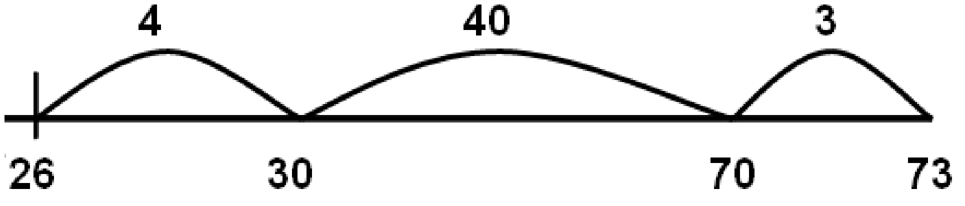
Number lines, however, make the calculation easy.

73 – 26 =



The use of number lines builds on the understanding of subtraction as difference or as complementary addition.

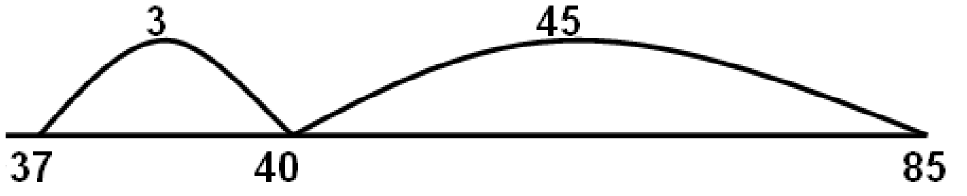
The jumps on this first number line are in tens and ones. This is a good starting point as it builds on the daily counting that children will be doing, including counting on in tens and ones from any number. It also means that calculating how many you have jumped altogether is easy. Of course, children may choose to do different jumps.



73 – 26 = 47

When they are confident with this stage, pupils can reduce the number of steps.

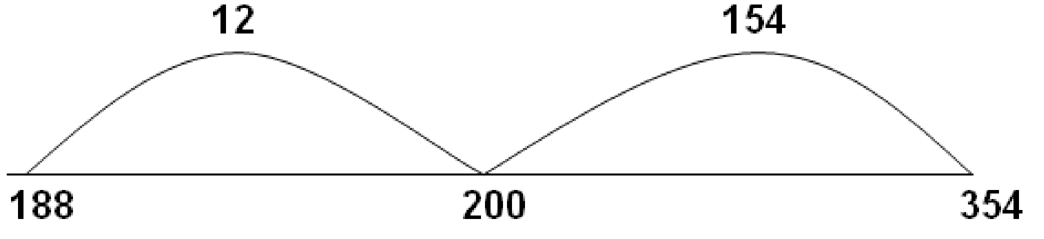
85 – 37 =



85 – 37 = 48

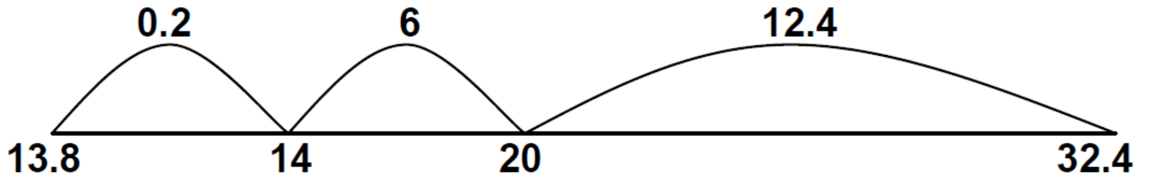
The above method can be extended to larger numbers by using complements to 100.

354 – 188 =



Subtraction of decimals is just as simple using the number line.

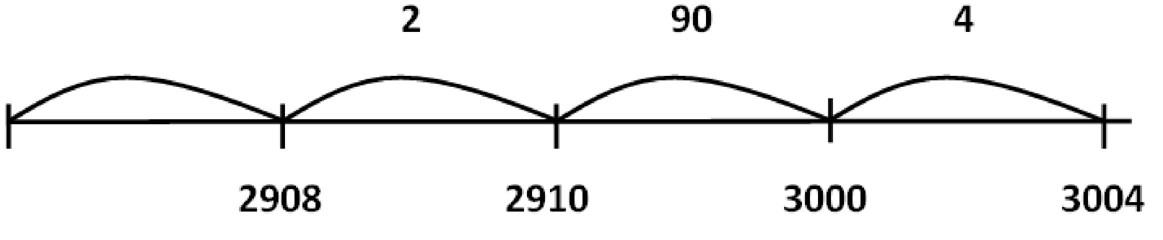
32.4 – 13.8



32.4 – 13.8 = 18.6

Children will still encounter calculations where it’s equally sensible to count back.

3004 – 96



Answer: 3004 – 96 = 2908

Counting on for Subtraction

This method requires the children to start with the number you are subtracting and count up, doing this in small steps. When counting up count to the nearest ten, the nearest hundred, then in hundreds to the target number, after that count up in tens and finally ones to the target number is reach. The number you counted will give you the answer/difference.

9 3 2
- 4 5 7

3	460
4 0	500
4 0 0	900
3 0	930
2	932

The figure on the right is a record of the number you have counted upto.

Vertical calculation for subtraction

In Key Stage 2 children will begin to use vertical calculations for subtraction that includes the expanded vertical method and compact decomposition.

Vertical calculation for subtraction can create real difficulties for both children and teachers. It’s easy to think that teaching children to remember a process, perhaps developed through the use of place value resources, will work. Some children may be able to remember this, but, even if they do, learning without understanding is never a basis for future development.

874 – 523 becomes

$$\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \\ \hline \end{array}$$

Answer: 351

The first step is to subtract 2 from 7 which is possible, but the answer would be negative. Partition the 30 into 20 and 10. Now you can put the 10 with the 2, so you have enough to subtract 7 without giving a negative answer.

The next step is to subtract 20 from 50. Again this would give a negative answer, so partition the 900 into 800 and put the 100 with the 20.

The final step is to subtract 50 from 120, which gives 70, and then subtract 800 from 400, leaving 400.

You can check that the answer is correct by adding the answer to 457: 457 + 475 = 932

Multiplication

Children’s first recording in multiplication will be by placing objects in arrays and counting in steps on number lines from zero.

In Year One children will look at number patterns and sequences whilst concentrating on counting in 2s, 5s and 10s (Some children will have been introduced to this concept at the end of Reception depending of their stage of development).

Concepts of multiplication develop using doubling and counting in steps, and are extended using number lines and number sentences, which will continue to be the main methods of recording.

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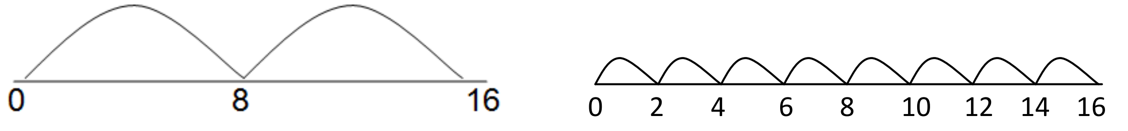
8 x 2 = 16



In Key Stage 1 and Key Stage 2 the recall of multiplication facts mentally is important in order to help children with all calculations in many different contexts.

In Key Stage 2, once pupils begin to multiply one-digit by two-digit numbers this will be by using partitioning.

8 x 23 = (8 x 10) + (8 x 10) + (8 x 3) = 80 + 80 + 24 = 184

This leads to the grid method of multiplication:

8 x 23 =

x	**8**
20	160
3	24

The final step is to add 160 + 24 to give you the answer 184

The grid method can then be used for 2-digit by 2-digit multiplication. At first just use numbers between 11 and 19. For instance try 16 x 14:

x	**10**	**6**	
10	100	60	= 160
4	40	24	= 84

The final step is to add 140 + 84 to give you the answer 224.

Later children can move on to other 2-digit numbers and decimals. For instance try 66 x 34:

x	60	6
30	1800	180
4	240	24
	= 2040	= 204

The final step is to add 2040 + 204 to give you the answer 2244.

Moving children to a vertical multiplication calculation needs to be done with care, ensuring that they understand what they are doing and why they are doing it. To start with it's important that all the steps that would occur in the grid method are replicated in the vertical one.

	2 3	
x	**8**	
	2 4	8 x 3
	1 6 0	20 x 8
	1 8 4	

Division

In the Foundation Stage division is taught through activities such as sharing objects.
In Key stage one, as with multiplication, division is recorded with objects, arrays, number lines or number sentences and using recall of multiplication facts.

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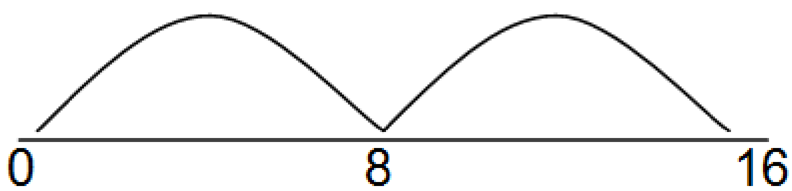
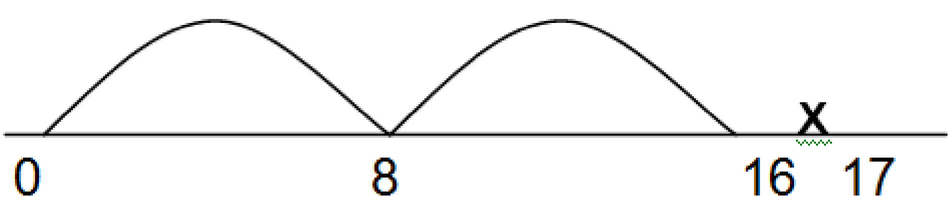
$16 \div 8 = 2$



Start at zero and count in 8s until you get to 16. That's two eighths.

Calculations with remainders in the quotient are also recorded on a number line.

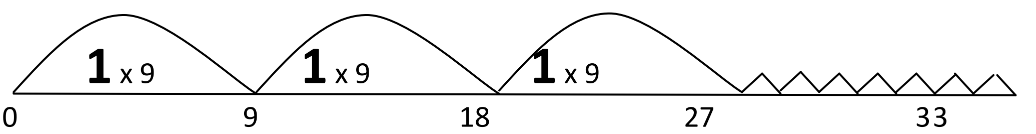


Start at zero and count in 8s until you get to 16. Then there is 1 more to get to 17, so there are 2 jumps of 8 and 1 left over (remainder). It's important that the remainder is never recorded as a jump as the jumps show how many eights have been made. Using a cross for each number left over tends to work well.

Arrays can be used to divide numbers and show remainders.
11 ÷ 2 = 5r1

* *
* *
* *
* *
* *
*

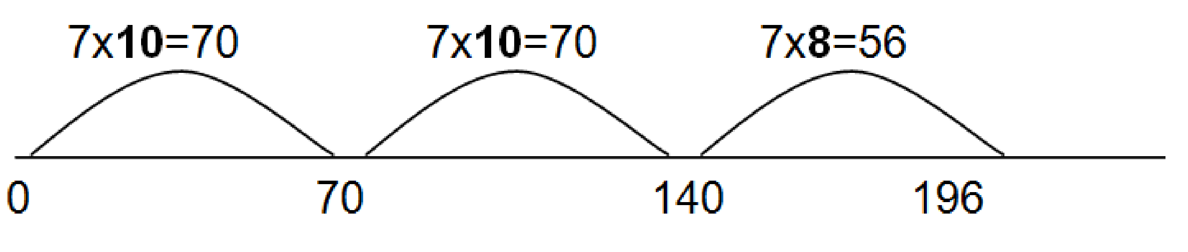
Another method is known as the WIK method and requires children to use their existing knowledge to help answer questions.
33 ÷ 9 =
What I Know (WIK)
2 x 9 = 18
4 x 9 = 36
10 x 9 = 90



33 ÷ 9 = 3 with 6 left over. (3r6)

In Key Stage 2, when children are dividing numbers which are more than 10 times the divisor it becomes useful to work with multiples of the divisor. In this example children would count in steps of 70, showing 7 ten times equals 70, then deciding how to do the next step of 56 ÷ 7. It could be one jump of 7 eight times, or could be smaller jumps of 7, 14, 21 and so on until the 196 is reached.

196 ÷ 7 = 28



In Year 4 children use the chunking method. They use knowledge of times tables including multiplying multiples of 10.

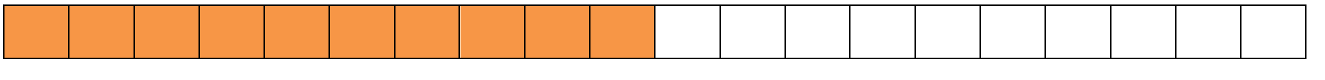
6	)1	9	6					
-	1	2	0	(2	0	x	6)	
		7	6					
-		6	0	(1	0	X	6)	
		1	6					
-		1	2	(	2	X	6)	
			4					
	3	2	r	4				

Fractions

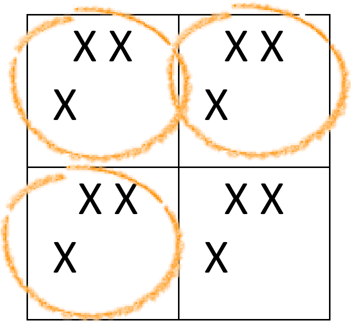
Early Years builds the foundations of understanding fractions through problem solving involving halves and sharing in a practical context.

In Key Stage 1, children need to visualise the question by first using practical activities to build an understanding. For example they may learn to recognise and find a half, as one of two equal parts of an object, by sharing or by folding a shape. They will then move on to finding quarters using similar practical methods.

Once confident with the concept of equal parts they will apply their knowledge to numbers up to 20 recording their work pictorially.

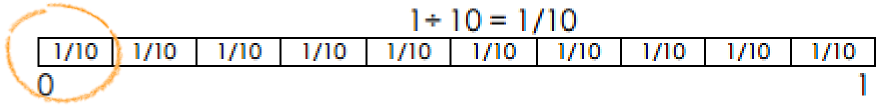


Continuing on, children will find, name and write fractions as a length, shape, set of objects or quantity. They use their knowledge of unit and non-unit fractions of shapes to find fractions of quantities. They relate this to find fractions of a length e.g. 2/4 of 100cm = Children need to relate finding a quarter to halving and halving again.

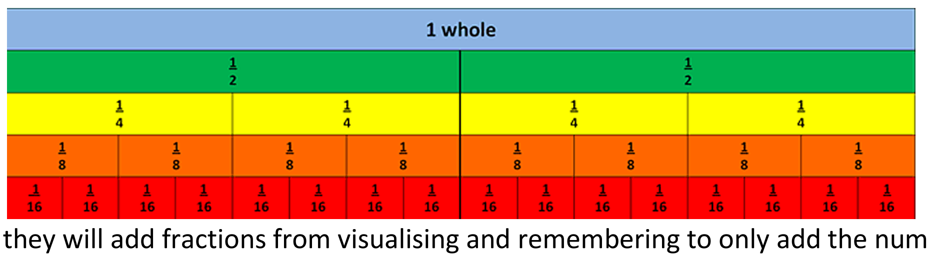


This diagram would give the children an example of sharing a quantity (12) into quarters. Then by each circling the section representing the numerator, we could show ¾.

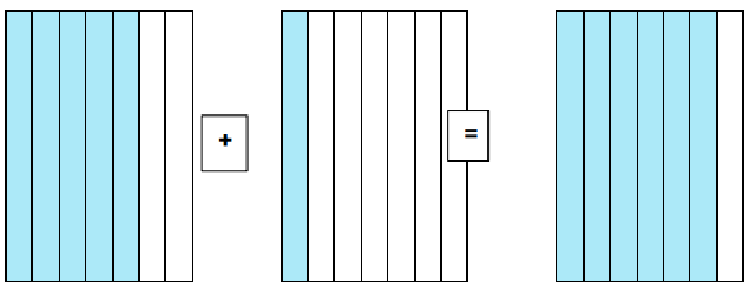
Moving into Key Stage 2, the children count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one digit numbers or quantities by 10.



Moving onto equivalents, children use practical means to find equivalent fractions with small denominators, they then compare and order unit fractions, and fractions with the same denominators E.g. 1/3 = 3/9 because 3 x 3 = 9.



Additionally they will add fractions from visualising and remembering to only add the numerator.



$$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$$

By Year 4, the children extend their knowledge into 100ths. They then move onto solving problems involving increasingly harder fractions to calculate quantities, such as different denominators using their understanding of equivalences, and problems where the total makes a quantity whole.

$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = 1\frac{1}{4}$