

Mental Calculation Policy (to match New National Mathematics Curriculum)

This document presupposes that you wish to teach calculation with understanding, and not just as a process that is to be remembered. The Calculation Policy clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports the development of mathematical concepts.

The Aims of the Curriculum:

The national curriculum for mathematics aims to ensure that all pupils:

become **fluent** in the fundamentals of mathematics, including through varied and frequent practise with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Recording

Recording is developed in a range of ways, see below. Although initially they will be developed in this order, once a way of recording, such as 'by showing real objects', is in place, that will continue to be used throughout the Primary years when needed. In EYFS most recording will be by showing real objects.

Development of recording:

- by showing real objects
- by photographing or drawing the calculation activity
- counting on a number line
- a practical calculation activity on a number line
- a number bond on a number line
- a mental calculation on a number line
- a practical activity as a number sentence
- a number bond as a number sentence
- a mental calculation as a number sentence

Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

Children are encouraged to give answers in full sentences to help them fully understand the maths concepts they are learning.

Teaching for Mastery: White Rose Planning

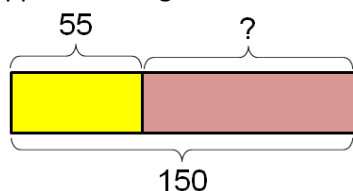
These planning overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- Have number at their heart. A large proportion of time is spent reinforcing number to build competency.
- Ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- Ensure students have the opportunity to stay together as they work through the schemes as a whole group.
- Provide plenty of time to build reasoning and problem solving elements into the curriculum.

Concrete – Pictorial – Abstract

- We believe that all students, when introduced to a key new concept, should have the opportunity to build competency in this topic by taking this approach.
- **Concrete** – students should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.
- **Pictorial** – students should then build on this concrete approach by using pictorial representations. These representations can then be used to reason and solve problems.
- **Abstract** – with the foundations firmly laid, students should be able to move to an abstract approach using numbers and key concepts with confidence.



An example of a bar modelling diagram used to solve problems.

Progression in calculation

Addition

Children begin calculation purely with practical activities using objects such as beads, playdough, bears and puzzles. Over time they learn to record these activities in a way that makes sense to them. This will be by showing or taking photographs of the equipment they have used, leading to drawings of what they did.

For instance, with the practical activity - I have 3 sweets, then I get one more. The child draws the sweets. They may draw 3 sweets and then another. They may just draw 4 to start with.

() () () ()

They won't draw 3, then 1, then 4, nor should they be expected to at this stage.

() () () () = () () () () doesn't make much sense. You either have 3 and 1 or you have 4. You never have both.

This means that any recording of the format $3+1=4$ is very unhelpful and is not based on their experience but on an abstract recording method.

When pupils are ready to record numerals (possibly during the summer term in reception but mainly in year 1) they may begin to record the above example with numbers as well:

() () () ()
3 **1**

or just as

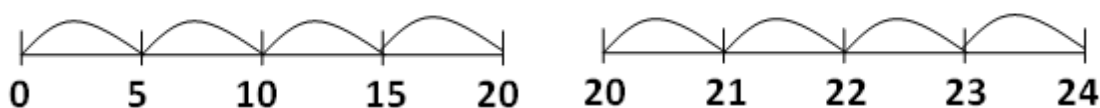
() () () ()
4

When the children are ready they will start to record they work in number sentences. For example

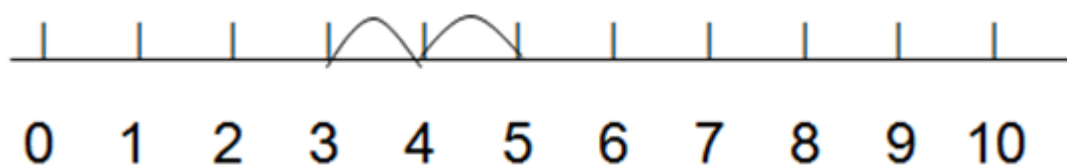
$$3 + 1 = 4$$

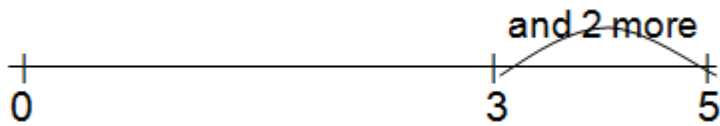
As well as using objects, pupils will begin to use number lines both as practical equipment that makes the calculation transparent and as ways to record what they did.

At first children will record their counting on number tracks, later moving to recording of calculation on a number line.

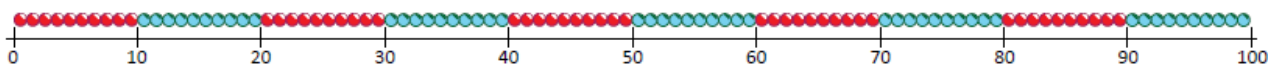
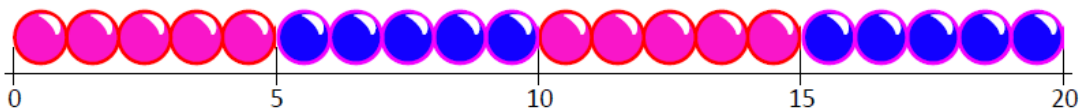


Pupils will use numbered number lines to record jumps, for example for $3+2$, before recording on blank number lines.

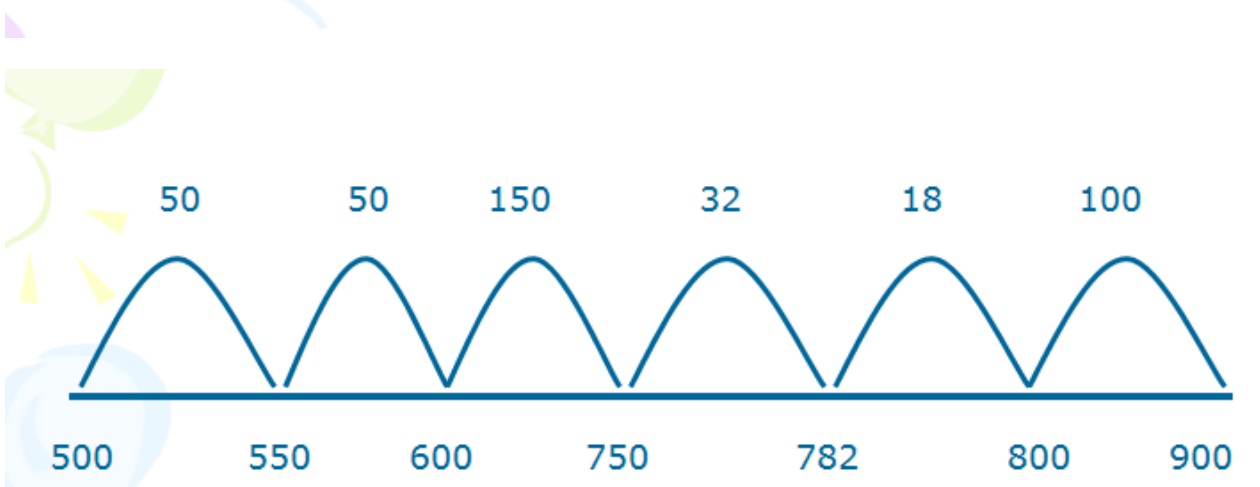
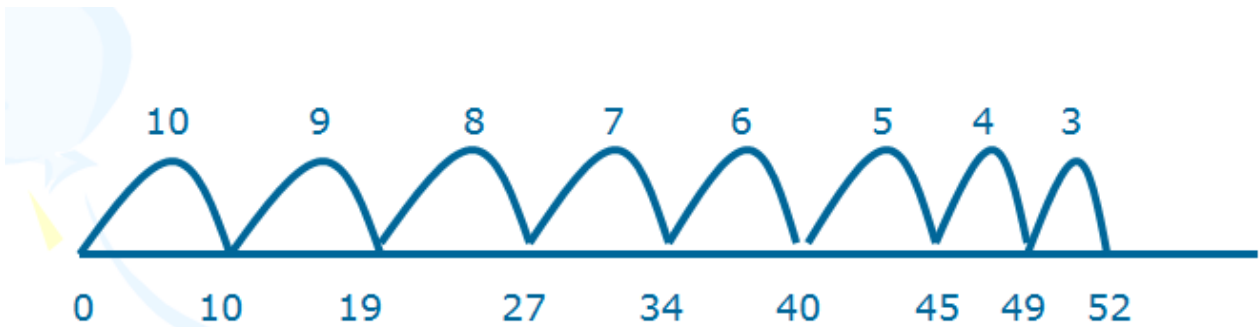




As well as using number tracks and number lines in Reception and Year 1 Children are able to use a physical bead string at the same time as the printed beaded number line, counting the beads as they move them, thus reinforcing the calculation they are making. They are great for finding one more / one less, counting in 2s, 5s and 10s, addition and subtraction.



By the end of Y1 children should be confident using number lines to 'experiment' with numbers. These examples are from average and above Y1 children at the end of the year. They have counted in different steps, and have seen their teacher record the numbers they have counted. They believe that maths is about experimenting with numbers and trying things out rather than just finding the right answer.



Recording number sentences

Before pupils move to recording $3+1$ they will need lots of experience of practical addition, and an ability to respond to mathematical vocabulary practically. For instance, if you ask a child to show you 5 and 2 more, or 3 plus 1, or 1 add 4, they can use the teddies, counters or number tracks to show you. They will also be developing their use of mathematical vocabulary to explain what they have done.

From this it will be possible to develop an understanding of the + sign, which will enable pupils to begin to record in the form $5+2$.

Pupils then need to understand the concept of equality before using the = sign. This means they can see an example such as $7=6+1$, or $5=5$, as well as the more common arrangement $3+1=4$, and know that it makes sense.

In Year 1 pupils will still work practically with equipment and real objects, but now can record their explanation of what they have done as a conventional number sentence:

$3 + 14 = 17$ $17 = 14 + 3$ $17 - 3 = 14$ $3 + 14 = 14 + 3$ and so on.

However, pupils will still record with objects, drawings and number lines **on a frequent basis**, and whenever they are learning new concepts or starting to use a wider range of numbers they will need to return to using these easily understood and explained methods of recording.

Pupils need to develop their use of jottings to support mental calculation. These jottings may be as drawings, number lines or number sentences.

In KS1 and KS2, once children have an understanding of place value in 2-digit numbers, in other words they are convinced that 23 is 20 and 3, or 59 is 50 and 9, they can begin to use partitioning in their mental calculations.

Partitioning

Partitioning may be recorded in a number of ways, such as:

$$\begin{aligned} 36 + 45 &= 30 + 40 + 6 + 5 \\ &= 70 + 11 \\ &= 81 \end{aligned}$$

$$\begin{aligned} 536 + 245 &= 500 + 200 + 30 + 40 + 6 + 5 \\ &= 700 + 70 + 11 \\ &= 781 \end{aligned}$$

or

$$\begin{aligned} 36 + 45 &= 36 + 40 + 5 \\ &= 76 + 5 \\ &= 81 \end{aligned}$$

The important thing to consider when children are recording partitioning is that they record how they thought about the numbers, and don't all try to do it the same way. This is not about finding lots of ways to record, but of recording what makes sense to a child.

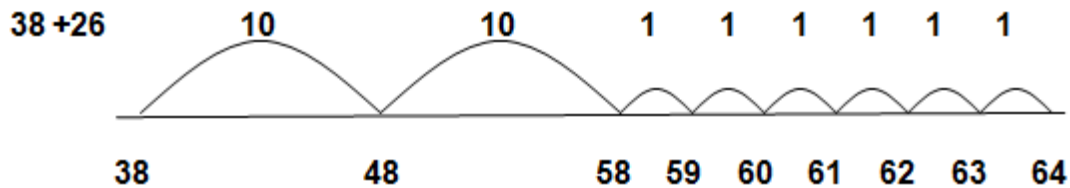
Partitioning is also an appropriate strategy for larger numbers, eventually including decimals.

Partitioning using number lines

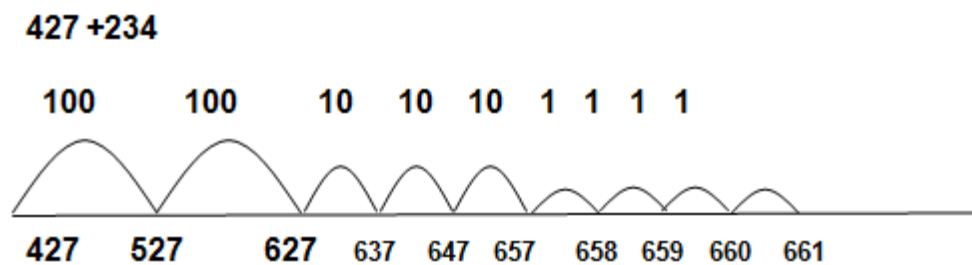
Key understanding – A number line is a tool, not a rule.

Children partition numbers to count on, mainly in multiples of 100, 10 or 1, on a number line. Number lines will be used for calculations right through Key Stage 2.

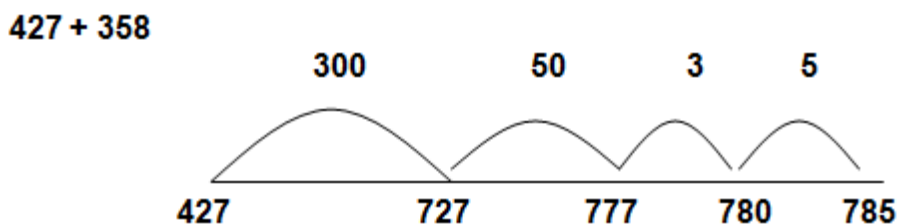
Initial attempts may be a little slow as children choose easy numbers to count on:



What matters, however, is that children make their own choices of which numbers to use and that they use their understanding of number and place value to find a way that works for them. This may continue into 3 and 4-digit numbers for some children.

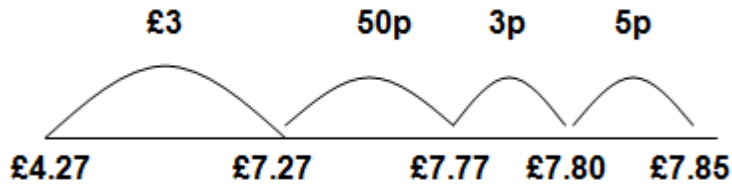


As they become more confident, children start to jump in multiples of 100, 10 and 1. They use their own choice of numbers, doing any jumps on the number line, in steps of 100, 10, 1 or multiples of these, depending on their mental strategies and ability.

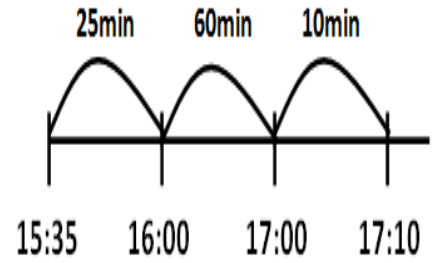


Children need to develop understanding of calculation in a range of contexts, for instance measures, including money and time.

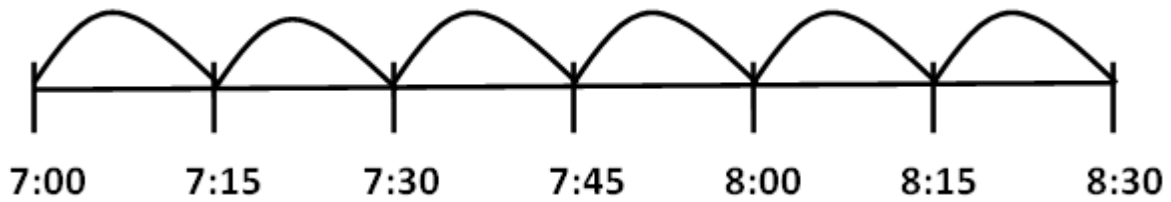
$£4.27 + £3.58$



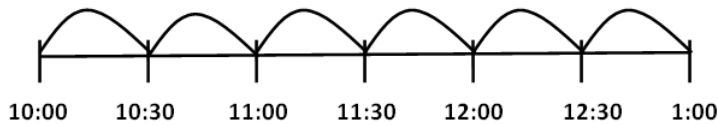
It is 15:35. What will the time be in 95 minutes?



Time is particularly difficult, and at first children will use number lines to record counting in steps of hours or minutes.



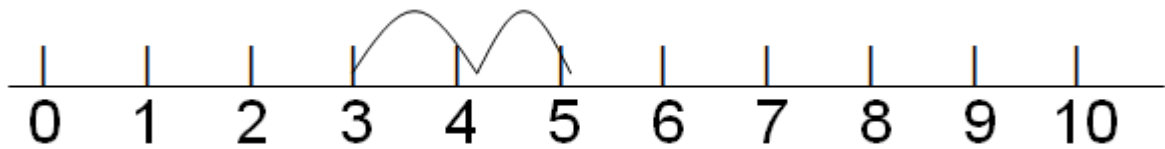
Counting across boundaries is particularly important.



Subtraction

As with addition, subtraction is initially recorded as drawing the result of a practical activity, moving on to record this using numbers, on number tracks or lines or as number sentences. Initially number tracks or lines will be used to subtract small numbers such as $5 - 2$.



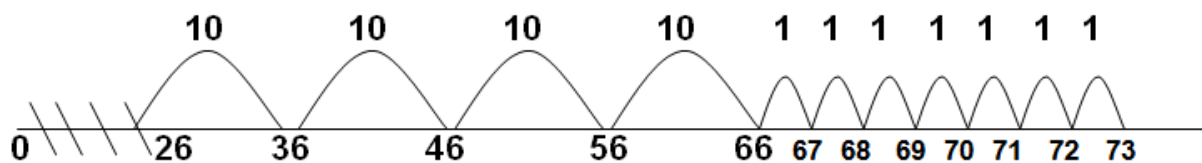


When pupils move on to use jottings the number line will become especially important. Jottings as number sentences are less useful for subtraction as partitioning cannot generally be used. In the example $73 - 26$ it is possible to start with $70 - 20$, but $3 - 6$ is less useful!

Key understanding – Pupils need to realise that partitioning is not appropriate for subtraction.

Number lines, however, make the calculation easy.

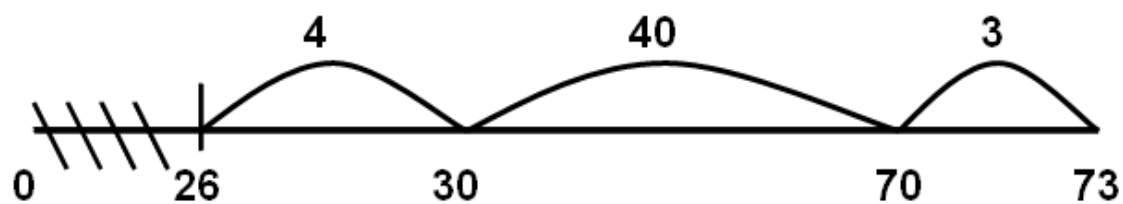
$$73 - 26 =$$



Key understanding – Putting the zero on a number line for subtraction and crossing out what has been subtracted makes the subtraction obvious.

You'll notice that there is a zero placed on the number line. This helps to stop children writing the 73 on the left hand side of the number line, but more importantly enables you to cross out and 'take away' the 26. It makes it easier to understand that this is a subtraction, and you are counting on to find out how many are left. So this use of number lines builds on the understanding of subtraction as difference or as complementary addition.

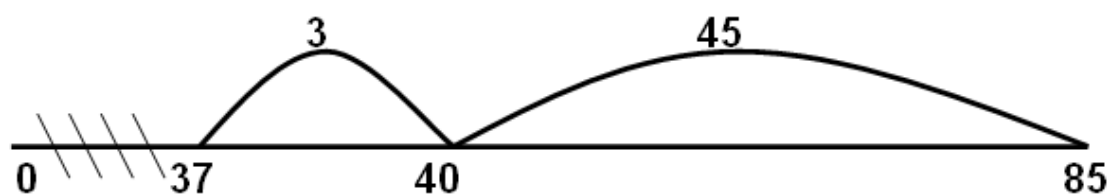
The jumps on this first number line are in tens and ones. This is a good starting point as it builds on the daily counting that children will be doing, including counting on in tens and ones from any number. It also means that calculating how many you have jumped altogether is easy. Of course children may do different jumps.



$$73 - 26 = 47$$

When they are confident with this stage, pupils can reduce the number of steps.

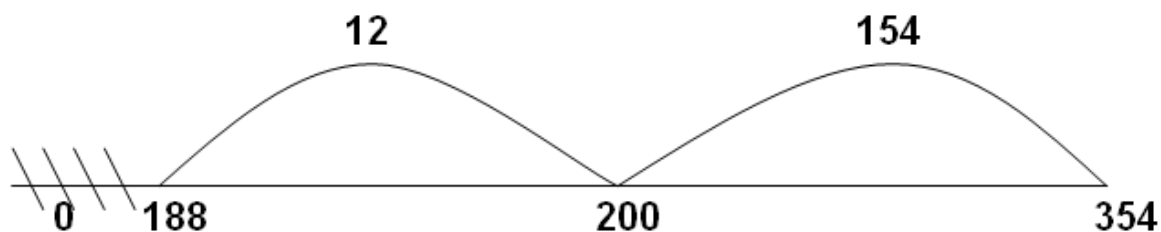
$$85 - 37 =$$



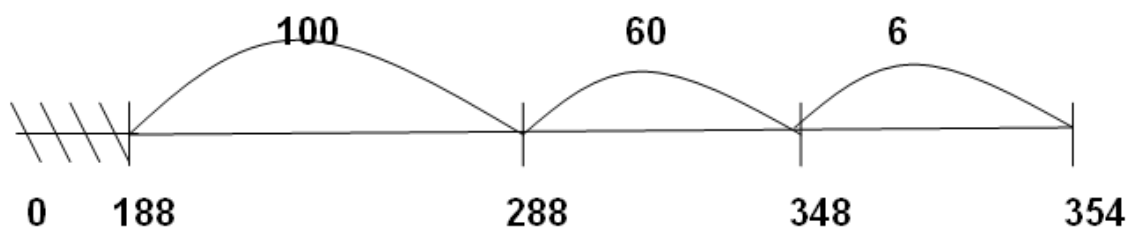
$$85 - 37 = 48$$

The above method can be extended to larger numbers by using complements to 100.

$$354 - 188 =$$



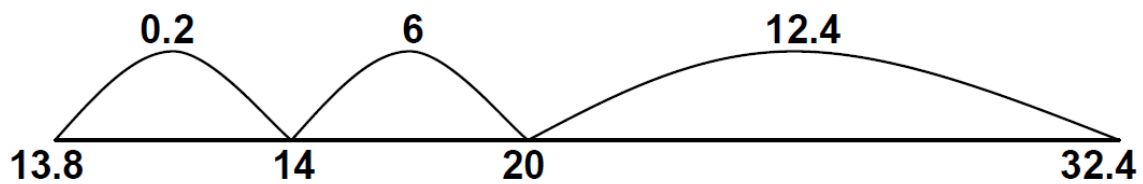
Children may still choose to count in multiples of 100, 10 and 1.



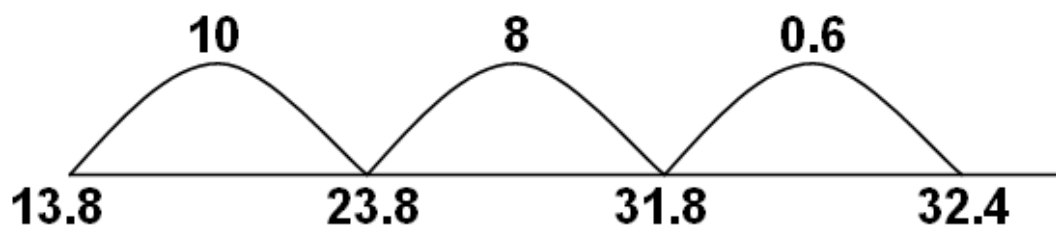
$$354 - 188 = 166$$

Subtraction of decimals is just as simple using the number line.

$$32.4 - 13.8$$



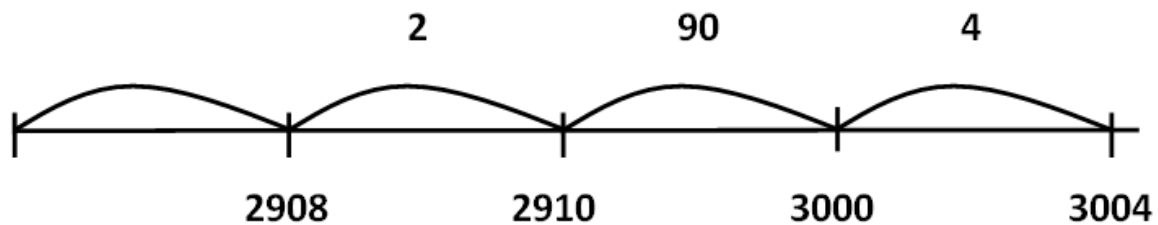
$$32.4 - 13.8 = 18.6$$



$$32.4 - 13.8 = 18.6$$

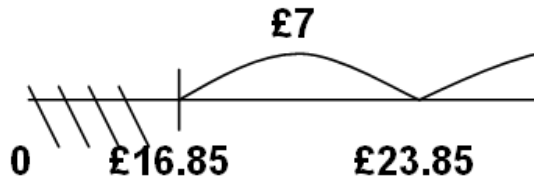
Children will still encounter calculations where it's equally sensible to count back.

$$3004 - 96$$

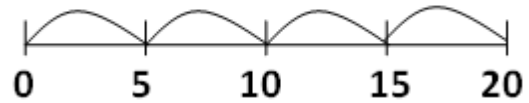


$$3004 - 96 = 2908$$

Children continue to use number lines for subtraction calculations in a range of contexts, such as time, money, mass, length and capacity.



$$£24.53 - £16.85 = £7.68$$



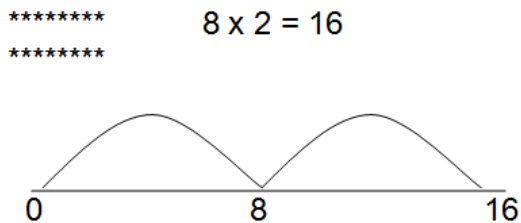
Multiplication

Children's first recording in multiplication will be by placing objects in arrays and counting in steps on number lines from zero.



In Year 1 children will look at number patterns and sequences whilst concentrating on counting in 2's, 5's and 10's.

Concepts of multiplication develop using doubling and counting in steps, and are extended using number lines and number sentences will continue to be the main methods of recording.



In Year 1 children will look at number patterns and sequences whilst concentrating on counting in 2's, 5's and 10's.

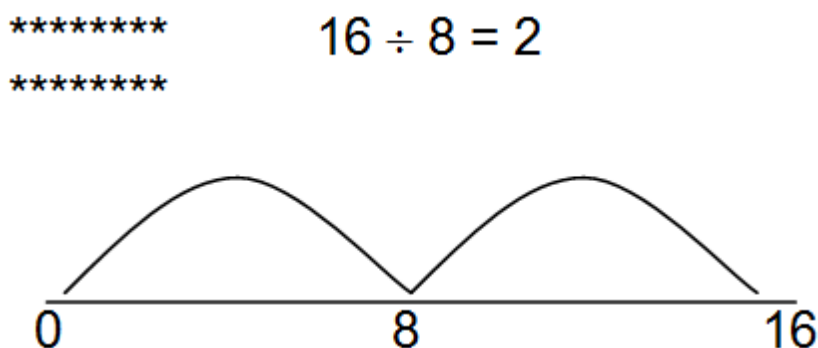
In KS1 and KS2 the recall of multiplication facts mentally is important in order to help children with all calculations in many different contexts.

In KS2, once pupils begin to multiply one-digit by two-digit numbers this will be by using partitioning.

$$8 \times 23 = 8 \times 10 + 8 \times 10 + 8 \times 3 = 80 + 80 + 24 = 184$$

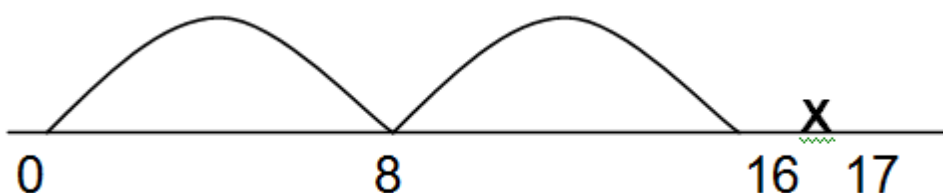
Division

As with multiplication, division is recorded with objects, arrays, number lines or number sentences and using recall of multiplication facts.



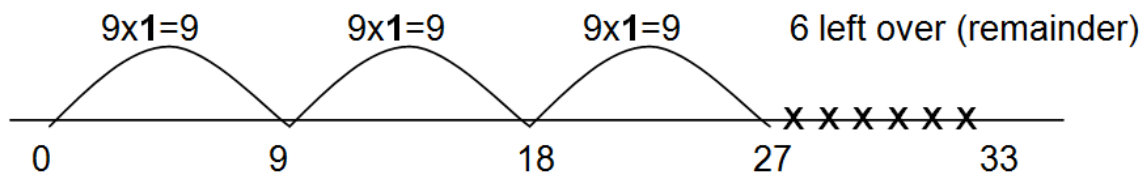
I start at zero and count in 8s until I get to 16. That's two eights.

Calculations with remainders in the quotient are also recorded on a number line.



I start at zero and count in 8s until I get to 16. Then there is 1 more to get to 17, so I have 2 jumps of 8 and 1 left over (remainder). It's important that the remainder is never recorded as a jump as the jumps show how many eights have been made. Using a cross for each number left over tends to work well.

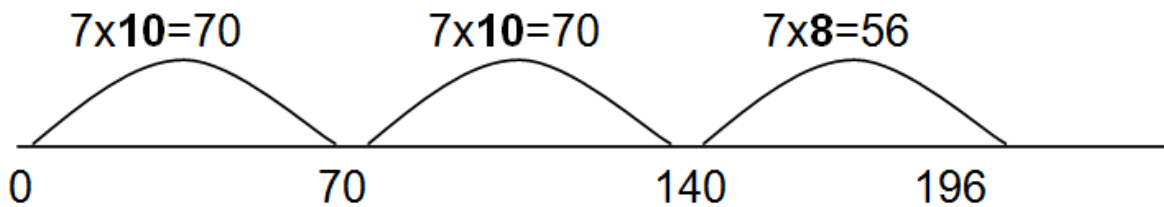
$$33 \div 9 =$$



$33 \div 9 = 3$ with 6 left over.

In KS2, when children are dividing numbers which are more than 10 times the divisor it becomes useful to work with multiples of the divisor. In this example children would count in steps of 70, showing 7 ten times equals 70, then deciding how to do the next step of $56 \div 7$. It could be one jump of 7 eight times, or could be smaller jumps of 7, 14, 21 and so on until the 196 is reached.

$$196 \div 7 =$$



Key understanding – Children must always record what happens in their own mind, and not try to guess and record what's in yours